**Problems on Double integrals over rectangles**

*Due date*: **Ashwin 1, 2081**

**1.** Use the Midpoint Rule to estimate the volume under *f* (*x*, *y*) = *x*2 + *y* and above the rectangle given by −1 ≤ *x* ≤ 3, 0 ≤ *y* ≤ 4 in the *xy*-plane. Use 4 subdivisions in the *x* direction and 2 subdivisions in the *y* direction.

**Ans.** Approx. 68

**2. (a)** Estimate the volume of the solid that lies below the surface *z* − *xy* and above the rectangle *R* = [0, 6] × [0, 4]. Use a Riemann sum with *m* = 3, *n* = 2 and take the sample point to be the upper right corner of each square.

**(b)** Use the Midpoint Rule to estimate the volume of the solid in part (a).

**Ans.** **(a)** Approx. 288 **(b)** Approx.144

**3.** If *R* = [0, 4] × [−1, 2], use a Riemann sum with *m* = 2, *n* = 3 to estimate the value of . Take the sample points to be (a) the lower right corners and (b) the upper left corners of the rectangles.

**Ans.** **(a)** Approx. –12 **(b)** Approx. –8

**4. (a)** Use a Riemann sum with *m* = *n* = 2 to estimate the value of ∫∫ , where *R* = [0, 2] × [0, 1]. Take the sample points to be upper right corners.

**(b)** Use the Midpoint Rule to estimate the integral in part (a).

**Ans.** **(a)** Approx. 0.990 **(b)** Approx.1.151

**5.** **(a)** Estimate the volume of the solid that lies below the surface *z* = 1 + *x*2 + 3*y* and above the rectangle *R* = [1, 2]×[0, 3]. Use a Riemann sum with *m* = *n* = 2 and choose the sample points to be lower left corners.

**(b)** Use the Midpoint Rule to estimate the volume in part (a).

**Ans.** **(a)** Approx.14.625 **(b)** Approx. 23.4375

**Evaluate the iterated integrals.**

**6.**

**Ans.** 40

**7.**

**Ans.** 40

**8.**

**Ans.** 4/3

**9.**

**Ans.** 84

**10.**

**Ans.** 4

**11.**

**Ans.** 1858/15

**12.**

**Ans.** –1

**13.**

**Ans.** 6 ln 6 − 2 ln 2 − 5 ln 5 ≈ 1*.*317

**14.**

**Ans.** *e*–1+*e*–*e*2–*e*4

**15.**

**Ans. (**4/3)(19 − 5√5) ≈ 10*.*426

**16.**

**Ans.**  + 16

**17.**

**Ans. (**1/2)(ln 3)(−2 + ln 48*)* ≈ 1*.*028

**18.**

**Ans.** –1/28

**Evaluate the double integral over the rectangular region *R*.**

**19.** ∫∫4*xy*3 *dA*; *R* = {(*x, y*): −1 ≤ *x* ≤ 1*,*−2 ≤ *y* ≤ 2}

**Ans.** 0

**20.** ∫∫ *dA*; *R* = {(*x, y*): 0 ≤ *x* ≤ 1*,* 0 ≤ *y* ≤ 1}

**Ans.** (3+ 4)

**21.** ∫∫*x* *dA*; *R* = {(*x, y*): 0 ≤ *x* ≤ 1*,* 2 ≤ *y* ≤ 3}

**Ans.**

**22.** ∫∫(*x* sin *y* − *y* sin *x*) *dA*; *R* = {*(x, y)* : 0 ≤ *x* ≤ *π/*2*,* 0 ≤ *y* ≤ *π/*3}

**Ans.** π

**Use a double integral to find the volume.** ■

**23.** The volume under the plane *z* = 2*x* + *y* and over the rectangle

*R* = {*(x, y)* : 3 ≤ *x* ≤ 5*,* 1 ≤ *y* ≤ 2}.

**Ans.** 19

**24.** The volume under the surface *z* = 3*x*3 + 3*x*2*y* and over the rectangle

*R* = {*(x, y)* : 1 ≤ *x* ≤ 3*,* 0 ≤ *y* ≤ 2}.

**Ans.** 172

**25.** The volume of the solid enclosed by the surface *z* = *x*2 and the planes

*x* = 0, *x* = 2, *y* = 3, *y* = 0, and *z* = 0.

**Ans.** 8

**26.** The volume in the first octant bounded by the coordinate planes, the plane *y* = 4, and the plane (*x/*3)+ (*z/*5)= 1.

**Ans.** 30

**27.** Find the average value of *f* (*x, y*)= *xy*2 over the rectangle [0*,* 8] × [0*,* 6].

**Ans.** 48

**28.** Find the average value of *f* (*x, y*)= *x*2 + 7*y* over the rectangle [0*,* 3] × [0*,* 6].

**Ans.** 24

**29.** Find the average value of *f* (*x, y*)= *y* sin *xy* over the rectangle [0*,* 1] × [0*, π/*2].

**Ans.** 1 – π

**30.** Find the average value of *f* (*x, y*)= *x*(*x*2 + *y*)1*/*2 over the rectangle [0*,* 1] × [0*,* 3].

**Ans.** (31 – 9)

**31.** Suppose that the temperature in degrees Celsius at a point (*x, y*)on a flat metal plate is

*T* (*x, y*)= 10 − 8*x*2 − 2*y*2, where *x* and *y* are in meters. Find the average temperature of the rectangular portion of the plate for which 0 ≤ *x* ≤ 1 and 0 ≤ *y* ≤ 2.

**Ans.** (14/3)°C

**32.** Show that if *f* (*x, y*)is constant on the rectangle *R* = [*a, b*] × [*c, d*], say *f* (*x, y*)= *k*, then

*f*ave = *k* over *R*.